

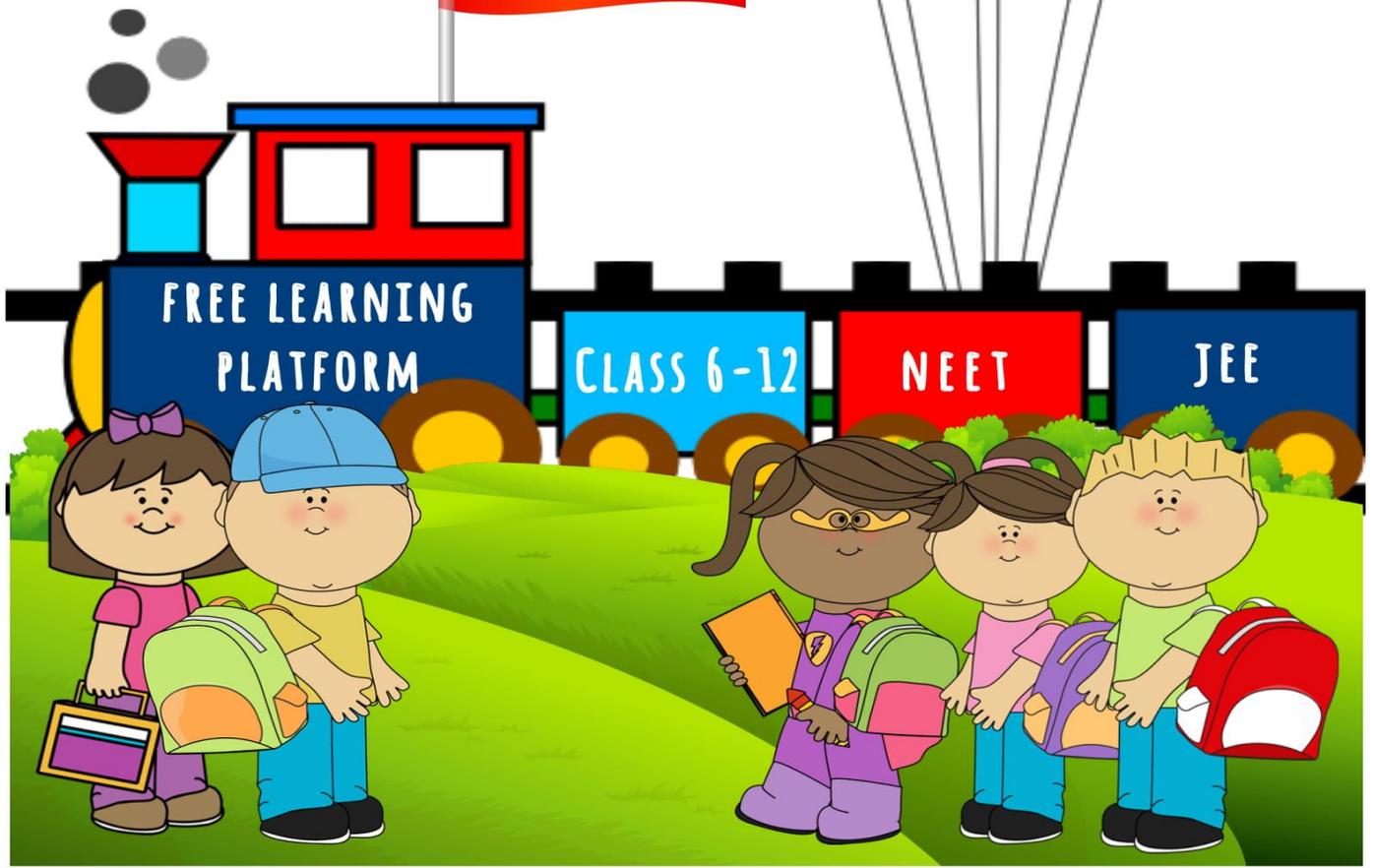
NCERT Solutions

Class 9 - Maths

Circles



EXAMFEAR
EDUCATION



Exercise 10.1

Question 1:

Fill in the blanks:

- (i) The centre of a circle lies in _____ of the circle. (exterior/ interior)
- (ii) A point, whose distance from the centre of a circle is greater than its radius lies in _____ of the circle. (exterior/ interior)
- (iii) The longest chord of a circle is a _____ of the circle.
- (iv) An arc is a _____ when its ends are the ends of a diameter.
- (v) Segment of a circle is the region between an arc and _____ of the circle.
- (vi) A circle divides the plane, on which it lies, in _____ parts.

Answer:

- (i) The centre of a circle lies in **interior** of the circle.
- (ii) A point, whose distance from the centre of a circle is greater than its radius lies in **exterior** of the circle.
- (iii) The longest chord of a circle is a **diameter** of the circle.
- (iv) An arc is a **semi-circle** when its ends are the ends of a diameter.
- (v) Segment of a circle is the region between an arc and **chord** of the circle.
- (vi) A circle divides the plane, on which it lies, in **three** parts.

Question 2:

Write True or False: Give reasons for your answers.

- (i) Line segment joining the centre to any point on the circle is a radius of the circle.
- (ii) A circle has only finite number of equal chords.
- (iii) If a circle is divided into three equal arcs, each is a major arc.
- (iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

Answer:

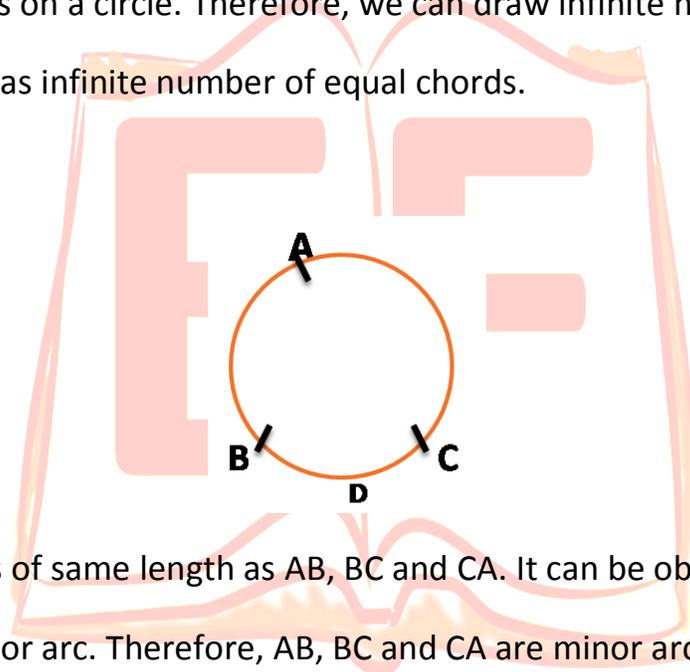
(i) True.

All the points on the circle are at equal distance from the center of the circle and this equal distance is called as radius of the circle.

(ii) False.

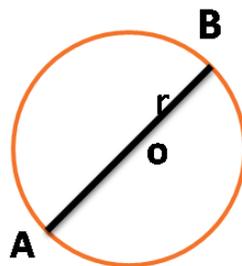
There are infinite points on a circle. Therefore, we can draw infinite number of chords of given length. Hence a circle has infinite number of equal chords.

(iii) False.



Let there are three arcs of same length as AB, BC and CA. It can be observed that the minor arc is BDC and CAB is a major arc. Therefore, AB, BC and CA are minor arcs of the circle.

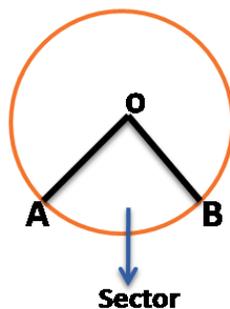
(iv) True.



Let AB be a chord which is twice as long as its radius. It can be observed that in this situation, our chord will be passing through the center of the circle. Therefore, it will be the diameter of

the circle.

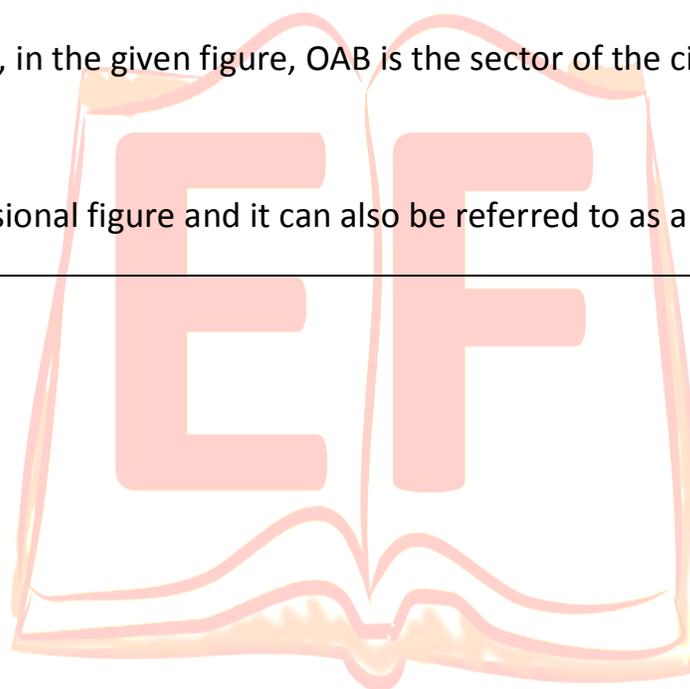
(v) False.



Sector is the region between an arc and two radii joining the center to the end points of the arc. For example, in the given figure, OAB is the sector of the circle.

(vi) True.

A circle is a two-dimensional figure and it can also be referred to as a plane figure.



Exercise 10.2

Question 1:

Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Answer:

Given : Two congruent circles with centres

O and O'. AB and CD are equal chords

of the circles with centres O and O' respectively.

To Prove: $\angle AOB = \angle COD$

Proof: In triangles AOB and COD,

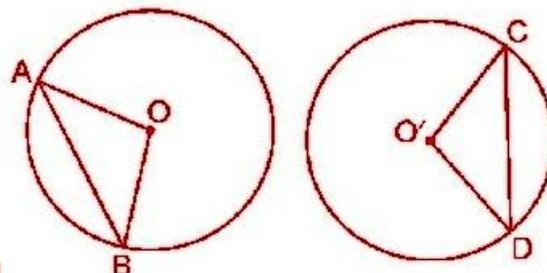
$AB = CD$ [Given]

$AO = CO'$ [Radii of congruent circle]

$BO = DO'$ [Radii of congruent circle]

$\triangle AOB \cong \triangle CO'D$ [by SSS axiom]

Hence, $\angle AOB \cong \angle CO'D$ [by CPCT]



Question 2:

Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Answer:

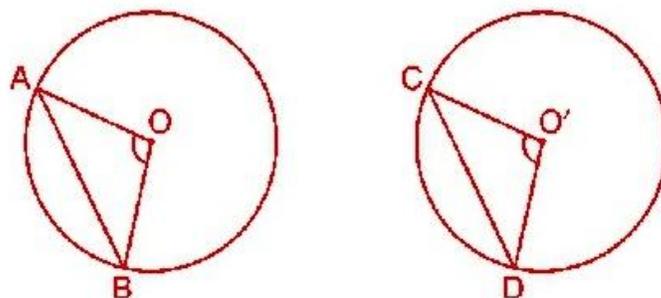
Given : Two congruent circles with

centres O and O'. AB and CD are

chords of circles with centre O

and O' respectively such that $\angle AOB = \angle CO'D$

To Prove: $AB = CD$



Proof: In triangles AOB and CO'D,

$AO = CO'$ [Radii of congruent circle]

$BO = DO'$ [Radii of congruent circle]

$\angle AOB = \angle CO'D$ [Given]

$\triangle AOB \cong \triangle CO'D$ [by SAS axiom]

Hence, $AB = CD$ [by CPCT]



Exercise 10.3

Question 1:

Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

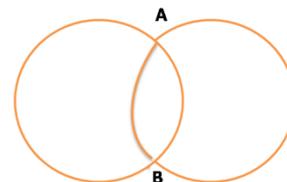
Answer:



No common point



One common point



Two common points

Hence, the maximum number of common point is two.

Question 2:

Suppose you are given a circle. Give a construction to find its centre.

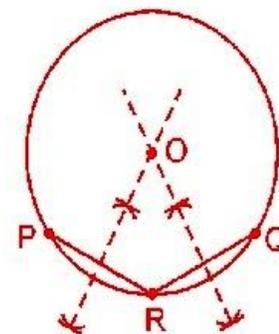
Answer:

Steps of Construction:

1. Take arc PQ of the given circle.
2. Take a point R on the arc PQ and draw chords PR and RQ.
3. Draw perpendicular bisectors of PR and RQ. These

perpendicular bisectors intersect at point O.

Hence, point O is the centre of the given circle.



Question 3:

If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Answer:

Given: AB is the common chord of two intersecting circles (O, r) and (O', r') .

To Prove: Centres of both circles lie on the perpendicular bisector of chord AB, i.e., AB is bisected at right angle by OO' .

Construction: Join AO , BO , AO' and BO' .

Proof: In $\triangle AOO'$ and $\triangle BOO'$

$$AO = BO \quad [\text{Radii of the circle } (O, r)]$$

$$AO' = BO' \quad [\text{Radii of the circle } (O', r')]$$

$$OO' = OO' \quad [\text{Common}]$$

$$\text{So, } \triangle AOO' \cong \triangle BOO' \quad [\text{by SSS congruency}]$$

$$\text{Hence, } \angle AOO' = \angle BOO' \quad [\text{by CPCT}]$$

Now in $\triangle AOC$ and $\triangle BOC$,

$$\angle AOC = \angle BOC \quad [\angle AOO' = \angle BOO']$$

$$AO = BO \quad [\text{Radii of the circle } (O, r)]$$

$$OC = OC \quad [\text{Common}]$$

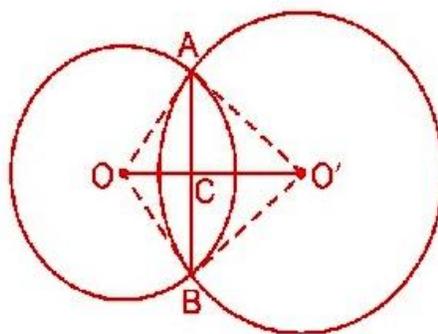
$$\text{So, } \triangle AOC \cong \triangle BOC \quad [\text{by SAS congruency}]$$

$$\text{Hence, } AC = BC \text{ and } \angle ACO = \angle BCO \dots\dots\dots(i) \quad [\text{by CPCT}]$$

$$\Rightarrow \angle ACO + \angle BCO = 180^\circ \dots\dots\dots(ii) \quad [\text{Linear pair}]$$

$$\Rightarrow \angle ACO = \angle BCO = 90^\circ \quad [\text{From equation 1 and 2}]$$

Hence, OO' lies on the perpendicular bisector of AB.



Exercise 10.4

Question 1:

Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Answer:

In $\triangle AOO'$,

$$AO^2 = 5^2 = 25$$

$$AO'^2 = 3^2 = 9$$

$$OO'^2 = 4^2 = 16$$

$$AO'^2 + OO'^2 = 9 + 16 = 25 = AO^2$$

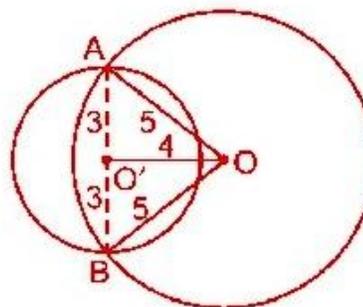
$$\Rightarrow \angle AO'O = 90^\circ \quad [\text{By converse of Pythagoras theorem}]$$

Similarly, $\angle BO'O = 90^\circ$.

$$\Rightarrow \angle AO'B = 90^\circ + 90^\circ = 180^\circ$$

$\Rightarrow AO'B$ is a straight line whose mid-point is O.

$$\Rightarrow AB = 3 + 3 = 6 \text{ cm.}$$



Question 2:

If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Answer:

Given: AB and CD are two equal chords of a circle which meet at E.

To prove: $AE = CE$ and $BE = DE$

Construction: Draw $OM \perp AB$ and $ON \perp CD$ and join OE.

Proof: In $\triangle OME$ and $\triangle ONE$,

$$OM = ON \quad [\text{Equal chords are equidistant}]$$

$OE = OE$ [Common]
 $\angle OME = \angle ONE$ [Each equal to 90°]

So, $\triangle OME \cong \triangle ONE$ [by RHS axiom]

$\Rightarrow EM = EN$ (i) [by CPCT]

Now $AB = CD$ [Given]

$\Rightarrow AB/2 = CD/2$

$\Rightarrow AM = CN$ (ii) [Perpendicular from centre bisects the chord]

Adding equation (i) and (ii), we get

$$EM + AM = EN + CN$$

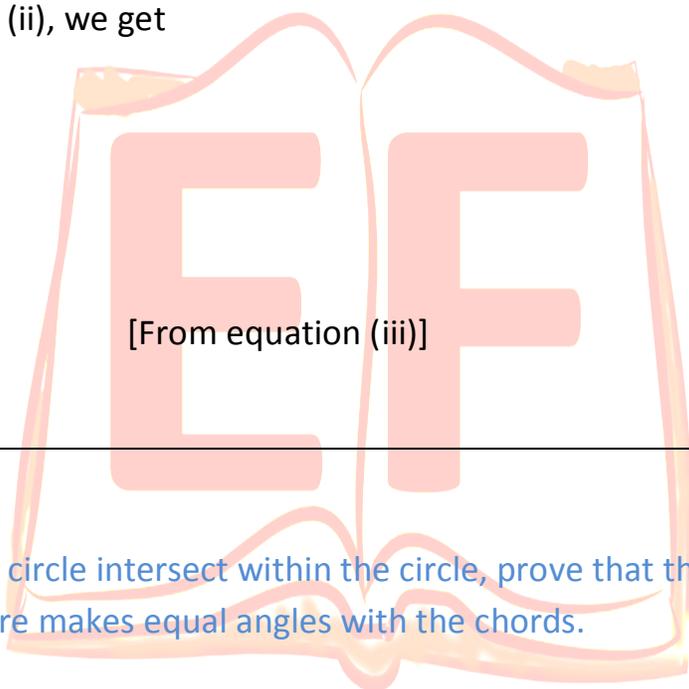
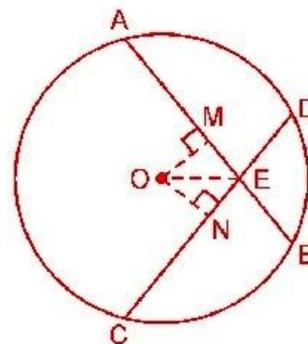
$\Rightarrow AE = CE$ (iii)

Now, $AB = CD$ (iv)

$\Rightarrow AB - AE = CD - AE$

[From equation (iii)]

$\Rightarrow BE = CD - CE$



Question 3:

If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Answer:

Given: AB and CD are two equal chords of a circle which meet at E within the circle and a line PQ joining the point of intersection to the centre.

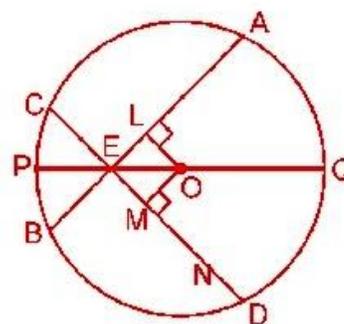
To Prove: $\angle AEQ = \angle DEQ$

Construction: Draw $OL \perp AB$ and $OM \perp CD$.

Proof: In $\triangle OLE$ and $\triangle OME$, we have

$OL = OM$ [Equal chords are equidistant]

$OE = OE$ [Common]



$$\angle OLE = \angle OME \quad [\text{Since each} = 90^\circ]$$

So, $\triangle OLE \cong \triangle OME$ [by RHS congruence]

$$\Rightarrow \angle LEO = \angle MEO \text{ [CPCT]}$$

Question 4:

If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB = CD$ (see Fig. 10.25).

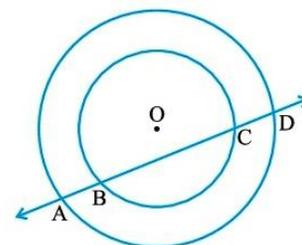


Fig. 10.25

Answer:

Given: A line AD intersects two concentric circles at A, B, C and D, where O is the centre of these circles.

To prove: $AB = CD$

Construction: Draw $OM \perp AD$.

Proof: AD is the chord of larger circle.

So, $AM = DM$ (i) [OM bisects the chord]

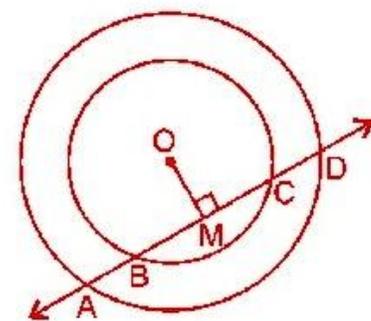
BC is the chord of smaller circle

So, $BM = CM$ (ii) [OM bisects the chord]

Subtracting equation (ii) from equation (i), we get

$$AM - BM = DM - CM$$

$$\Rightarrow AB = CD$$



Question 5:

Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Answer:

Let Reshma, Salma and Mandip be represented

by R, S and M respectively.

Now, Draw $OL \perp RS$.

$$OL^2 = OR^2 - RL^2$$

$$\Rightarrow OL^2 = 5^2 - 3^2$$

[$RL = 3$ m, because $OL \perp RS$]

$$\Rightarrow OL^2 = 25 - 9$$

$$\Rightarrow OL^2 = 16$$

$$\Rightarrow OL = \sqrt{16}$$

$$\Rightarrow OL = 4$$

Now, area of triangle ORS = $(1/2) * KR * OR$

$$= (1/2) * KR * 5$$

Also, area of $\Delta ORS = (1/2) * RS * OL$

$$= (1/2) * 6 * 4$$

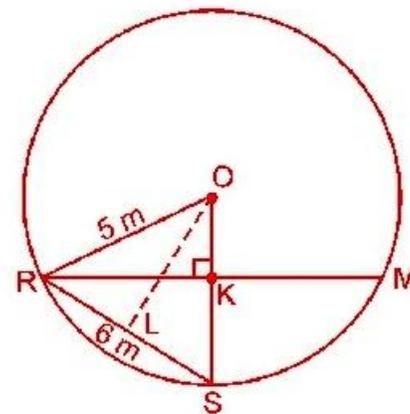
$$= 12 \text{ m}^2$$

Now, $(1/2) * KR * 5 = 12$

$$\Rightarrow KR = (12 * 2)/5$$

$$\Rightarrow KR = 24/5$$

$$\Rightarrow KR = 4.8 \text{ m}$$



Now, $RM = 2KR$

$$\Rightarrow RM = 2 * 4.8$$

$$\Rightarrow RM = 9.6 \text{ m}$$

Hence, distance between Reshma and Mandip is 9.6 m.

Question 6:

A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Answer:

Let Ankur, Syed and David be represented by A, S and D respectively.

Let $PD = SP = SQ = QA = AR = RD = x$

In $\triangle OPD$,

$$OP^2 = 400 - x^2$$

$$\Rightarrow OP = \sqrt{400 - x^2}$$

$$\Rightarrow AP = 2\sqrt{400 - x^2} + \sqrt{400 - x^2} \quad [\text{Since centroid divides the median in the ratio } 2 : 1]$$

$$\Rightarrow AP = 3\sqrt{400 - x^2}$$

Now, in $\triangle APD$,

$$PD^2 = AD^2 - AP^2$$

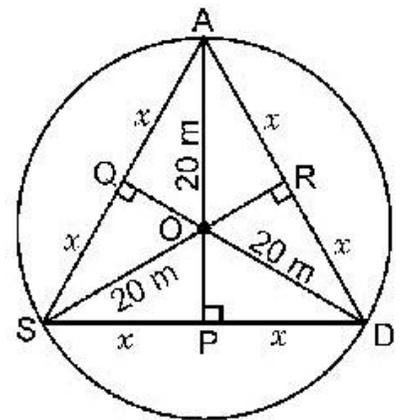
$$\Rightarrow x^2 = (2x)^2 - \{3\sqrt{400 - x^2}\}^2$$

$$\Rightarrow x^2 = 4x^2 - 9(400 - x^2)$$

$$\Rightarrow x^2 = 4x^2 - 3600 + 9x^2$$

$$\Rightarrow x^2 = 13x^2 - 3600$$

$$\Rightarrow 13x^2 - x^2 = 3600$$



$$\Rightarrow 12x^2 = 3600$$

$$\Rightarrow x^2 = 3600/12$$

$$\Rightarrow x^2 = 300$$

$$\Rightarrow x = \sqrt{300}$$

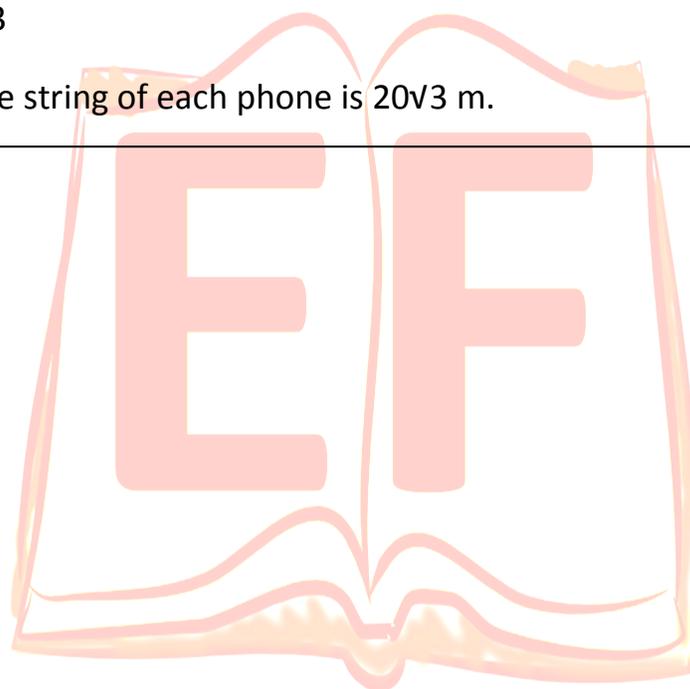
$$\Rightarrow x = 10\sqrt{3}$$

$$\text{Now, } SD = 2x = 2 * 10\sqrt{3} = 20\sqrt{3}$$

Hence, ASD is an equilateral triangle.

$$\text{So, } SD = AS = AD = 20\sqrt{3}$$

Hence, the length of the string of each phone is $20\sqrt{3}$ m.



Exercise 10.5

Question 1:

In Fig. 10.36, A, B and C are three points on a circle with centre O such that $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.

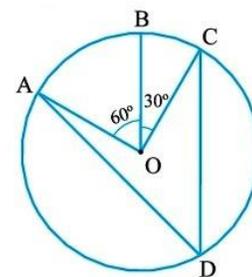


Fig. 10.36

Answer:

We have, $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$

Now, $\angle AOC = \angle AOB + \angle BOC$

$$= 60^\circ + 30^\circ$$

$$= 90^\circ$$

We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.

So, $2\angle ADC = \angle AOC$

$\Rightarrow \angle ADC = \angle AOC/2$

$$= 90^\circ/2$$

$\Rightarrow \angle ADC = 45^\circ$

Question 2:

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Answer:

We have, $OA = OB = AB$

Therefore, $\triangle OAB$ is an equilateral triangle.

$\Rightarrow \angle AOB = 60^\circ$

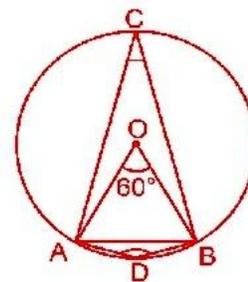
We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.

$$\text{So, } \angle AOB = 2\angle ACB$$

$$\Rightarrow \angle ACB = \angle AOB/2$$

$$= 60^\circ/2$$

$$\Rightarrow \angle ACB = 30^\circ$$



$$\text{Also, } \angle ADB = \text{reflex } \angle AOB/2$$

$$= (360^\circ - 60^\circ)/2$$

$$= 300^\circ/2$$

$$= 150^\circ$$

Question 3:

In Fig. 10.37, $\angle PQR = 100^\circ$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$

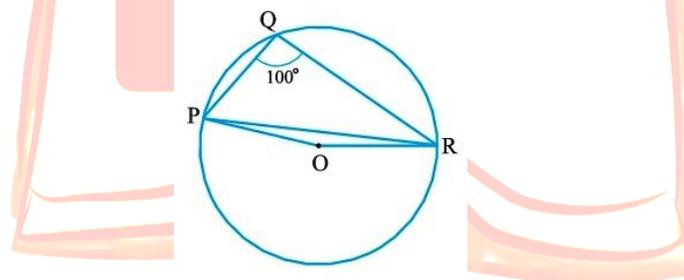


Fig. 10.37

Answer:

$$\text{Reflex angle POR} = 2\angle PQR$$

$$= 2 * 100^\circ$$

$$= 200^\circ$$

$$\text{Now, angle POR} = 360^\circ - 200^\circ = 160^\circ$$

$$\text{Also, } PO = OR \quad [\text{Radii of a circle}]$$

$$\angle OPR = \angle ORP \quad [\text{Opposite angles of isosceles triangle}]$$

In $\triangle OPR$,

$$\angle POR = 160^\circ$$

So, $\angle OPR = \angle ORP = 10^\circ$ [Angle sum property of a triangle]

Question 4:

In Fig. 10.38, $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$.

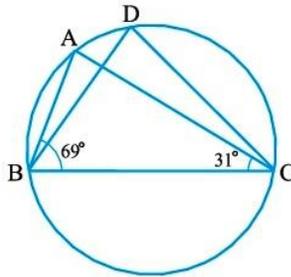


Fig. 10.38

Answer:

In $\triangle ABC$, we have

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ \quad \text{[Angle sum property of a triangle]}$$

$$\Rightarrow 69^\circ + 31^\circ + \angle BAC = 180^\circ$$

$$\Rightarrow 100^\circ + \angle BAC = 180^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - 100^\circ$$

$$\Rightarrow \angle BAC = 80^\circ$$

Also, $\angle BAC = \angle BDC$ [Angles in the same segment]

So, $\angle BDC = 80^\circ$

Question 5:

In Fig. 10.39, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. Find $\angle BAC$

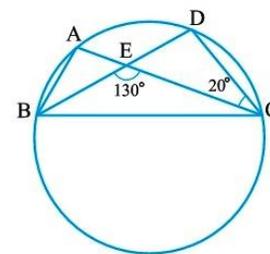


Fig. 10.39

Answer:

From the figure,

$$\angle BEC + \angle DEC = 180^\circ$$

[Linear pair]

$$\Rightarrow 130^\circ + \angle DEC = 180^\circ$$

$$\Rightarrow \angle DEC = 180^\circ - 130^\circ$$

$$\Rightarrow \angle DEC = 50^\circ$$

Now, in $\triangle DEC$,

$$\angle DEC + \angle DCE + \angle CDE = 180^\circ$$

[Angle sum property of a triangle]

$$\Rightarrow 50^\circ + 20^\circ + \angle CDE = 180^\circ$$

$$\Rightarrow 70^\circ + \angle CDE = 180^\circ$$

$$\Rightarrow \angle CDE = 180^\circ - 70^\circ$$

$$\Rightarrow \angle CDE = 110^\circ$$

Also, $\angle CDE = \angle BAC$

[Angles in same segment]

$$\Rightarrow \angle BAC = 110^\circ$$

Question 6:

ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC$ is 30° , find $\angle BCD$. Further, if $AB = BC$, find $\angle ECD$.

Answer:

From the figure,

$$\angle CAD = \angle DBC = 70^\circ \quad [\text{Angles in the same segment}]$$

Therefore, $\angle DAB = \angle CAD + \angle BAC$

$$= 70^\circ + 30^\circ$$

$$= 100^\circ$$

But, $\angle DAB + \angle BCD = 180^\circ$ [Opposite angles of a cyclic quadrilateral]

$$\text{So, } \angle BCD = 180^\circ - 100^\circ$$

$$= 80^\circ$$

Now, we have $AB = BC$

Therefore, $\angle BCA = 30^\circ$ [Opposite angles of an isosceles triangle]

Again, $\angle DAB + \angle BCD = 180^\circ$ [Opposite angles of a cyclic quadrilateral]

$$\Rightarrow 100^\circ + \angle BCA + \angle ECD = 180^\circ \quad [\text{Since } \angle BCD = \angle BCA + \angle ECD]$$

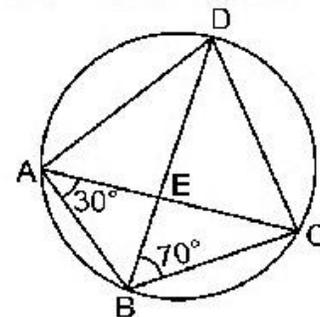
$$\Rightarrow 100^\circ + 30^\circ + \angle ECD = 180^\circ$$

$$\Rightarrow 130^\circ + \angle ECD = 180^\circ$$

$$\Rightarrow \angle ECD = 180^\circ - 130^\circ$$

$$\Rightarrow \angle ECD = 50^\circ$$

Hence, $\angle BCD = 80^\circ$ and $\angle ECD = 50^\circ$



Question 7:

If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Answer:

Given: ABCD is a cyclic quadrilateral, whose diagonals AC and BD are diameter of the circle
Passing through A, B, C and D.

To Prove: ABCD is a rectangle.

Proof:

In $\triangle AOD$ and $\triangle COB$,

$AO = CO$ [Radii of a circle]

$OD = OB$ [Radii of a circle]

$\angle AOD = \angle COB$ [Vertically opposite angles]

So, $\triangle AOD \cong \triangle COB$ [by SAS axiom]

Hence, $\angle OAD = \angle OCB$ [by CPCT]

But these are alternate interior angles made by the transversal AC, intersecting AD and BC.

So, $AD \parallel BC$

Similarly, $AB \parallel CD$.

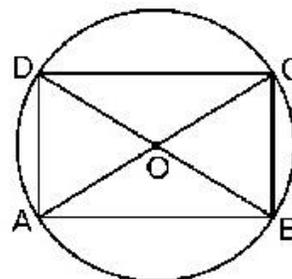
Hence, quadrilateral ABCD is a parallelogram.

Also, $\angle ABC = \angle ADC$ (i) [Opposite angles of a parallelogram are equal]

And, $\angle ABC + \angle ADC = 180^\circ$...(ii) [Sum of opposite angles of a cyclic quadrilateral is 180°]

$\Rightarrow \angle ABC = \angle ADC = 90^\circ$ [From equation (i) and (ii)]

So, ABCD is a rectangle. [A parallelogram one of whose angles is 90° is a rectangle]



Question 8:

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Answer:

Given: A trapezium ABCD in which $AB \parallel CD$ and $AD = BC$.

To Prove: ABCD is a cyclic trapezium.

Construction: Draw $DE \perp AB$ and $CF \perp AB$.

Proof:

In $\triangle DEA$ and $\triangle CFB$, we have

$$AD = BC \quad [\text{Given}]$$

$$\angle DEA = \angle CFB = 90^\circ \quad [DE \perp AB \text{ and } CF \perp AB]$$

$$DE = CF \quad [\text{Distance between parallel lines remains constant}]$$

$$\text{So, } \triangle DEA \cong \triangle CFB \quad [\text{by RHS axiom}]$$

$$\Rightarrow \angle A = \angle B \quad \dots\dots(i) \quad [\text{by CPCT}]$$

$$\text{and, } \angle ADE = \angle BCF \quad \dots\dots(ii) \quad [\text{by CPCT}]$$

$$\text{Since, } \angle ADE = \angle BCF \quad [\text{From equation (ii)}]$$

$$\Rightarrow \angle ADE + 90^\circ = \angle BCF + 90^\circ \quad \Rightarrow \angle ADE + \angle CDE = \angle BCF + \angle DCF$$

$$\Rightarrow \angle D = \angle C \quad \dots\dots(iii) \quad [\angle ADE + \angle CDE = \angle D, \angle BCF + \angle DCF = \angle C]$$

$$\text{So, } \angle A = \angle B \text{ and } \angle C = \angle D \quad \dots\dots(iv) \quad [\text{From (i) and (iii)}]$$

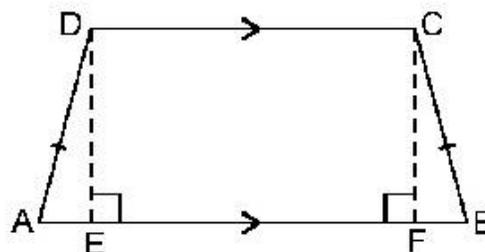
$$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ \quad [\text{Sum of the angles of a quadrilateral is } 360^\circ]$$

$$\Rightarrow 2(\angle B + \angle D) = 360^\circ \quad [\text{Using equation (iv)}]$$

$$\Rightarrow \angle B + \angle D = 180^\circ$$

$$\Rightarrow \text{Sum of a pair of opposite angles of quadrilateral ABCD is } 180^\circ.$$

$$\Rightarrow \text{ABCD is a cyclic trapezium}$$



Question 9:

Two circles intersect at two points B and C.

Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 10.40). Prove that

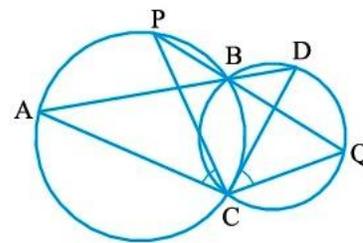


Fig. 10.40

$$\angle ACP = \angle QCD$$

Answer:

Given: Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.

To Prove: $\angle ACP = \angle QCD$.

Proof:

$$\angle ACP = \angle ABP \dots\dots(i) \quad [\text{Angles in the same segment}]$$

$$\angle QCD = \angle QBD \dots\dots(ii) \quad [\text{Angles in the same segment}]$$

$$\text{But, } \angle ABP = \angle QBD \dots\dots(iii) \quad [\text{Vertically opposite angles}]$$

From equation (i), (ii) and (iii), we get

$$\angle ACP = \angle QCD$$

Question 10:

If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

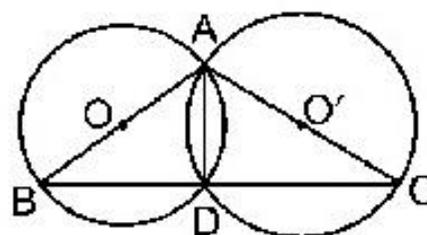
Answer:

Given: Sides AB and AC of a triangle ABC are diameters of two circles which intersect at D.

To Prove: D lies on BC.

Proof: Join AD

$$\angle ADB = 90^\circ \dots\dots(i) \quad [\text{Angle in a semicircle}]$$



Also, $\angle ADC = 90^\circ$ (ii)

Adding equation (i) and (ii), we get

$$\angle ADB + \angle ADC = 90^\circ + 90^\circ$$

$$\Rightarrow \angle ADB + \angle ADC = 180^\circ$$

\Rightarrow BDC is a straight line.

So, D lies on BC

Hence, point of intersection of circles lie on the third side BC.

Question 11:

ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Answer:

Given: ABC and ADC are two right triangles with common hypotenuse AC.

To Prove: $\angle CAD = \angle CBD$

Proof: Let O be the mid-point of AC.

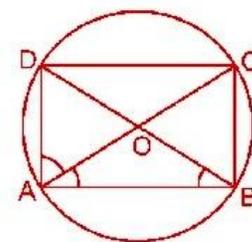
Then $OA = OB = OC = OD$

Mid point of the hypotenuse of a right triangle is equidistant from its vertices with O as centre and radius equal to OA, draw a circle to pass through A, B, C and D.

We know that angles in the same segment of a circle are equal.

Since, $\angle CAD$ and $\angle CBD$ are angles of the same segment.

Therefore, $\angle CAD = \angle CBD$.

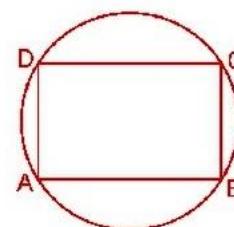


Question 12:

Prove that a cyclic parallelogram is a rectangle.

Answer:

Given: ABCD is a cyclic parallelogram.



To prove: ABCD is a rectangle.

Proof: $\angle ABC = \angle ADC$ (i) [Opposite angles of a parallelogram are equal]

But, $\angle ABC + \angle ADC = 180^\circ$... (ii) [Sum of opposite angles of a cyclic quadrilateral is 180°]

$\Rightarrow \angle ABC = \angle ADC = 90^\circ$ [From equation (i) and (ii)]

So, ABCD is a rectangle [A parallelogram one of whose angles is 90° is a rectangle]

Hence, a cyclic parallelogram is a rectangle.



Exercise 10.6 (Optional)

Question 1:

Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Answer:

Given : Two intersecting circles, in which OO' is the line of centres and A and B are two points of intersection.

To prove: $\angle OAO' = \angle OBO'$

Construction: Join AO, BO, AO' and BO'.

Proof: In $\triangle AOO'$ and $\triangle BOO'$, we have

$AO = BO$ [Radii of the same circle]

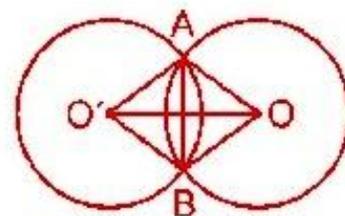
$AO' = BO'$ [Radii of the same circle]

$OO' = OO'$ [Common]

So, $\triangle AOO' \cong \triangle BOO'$ [by SSS axiom]

$\Rightarrow \angle OAO' = \angle OBO'$ [by CPCT]

Hence, the line of centres of two intersecting circles subtends equal angles at the two points of intersection.



Question 2:

Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Answer:

Let O be the centre of the circle and let its radius be r cm.

Draw $OM \perp AB$ and $OL \perp CD$.

Then, $AM = AB/2 = 5/2$ cm

and, $CL = CD/2 = 11/2$ cm

Since, $AB \parallel CD$, it follows that the points O, L, M are collinear and therefore, $LM = 6$ cm.

Let $OL = x$ cm. Then $OM = (6 - x)$ cm

Join OA and OC. Then $OA = OC = r$ cm.

Now, from right-angled $\triangle OMA$ and $\triangle OLC$, we have

$$OA^2 = OM^2 + AM^2 \text{ and } OC^2 = OL^2 + CL^2 \quad [\text{By Pythagoras Theorem}]$$

$$\Rightarrow r^2 = (6 - x)^2 + (5/2)^2 \dots\dots\dots(i)$$

$$\text{and } r^2 = x^2 + (11/2)^2 \dots\dots\dots(ii)$$

From equation 1 and 2, we get

$$\Rightarrow (6 - x)^2 + (5/2)^2 = x^2 + (11/2)^2$$

$$\Rightarrow 36 + x^2 - 12x + 25/4 = x^2 + 121/4$$

$$\Rightarrow -12x = 121/4 - 25/4 - 36$$

$$\Rightarrow -12x = 96/4 - 36$$

$$\Rightarrow -12x = 24 - 36$$

$$\Rightarrow 12x = -12$$

$$\Rightarrow x = 1$$

Substituting $x = 1$ in equation (i), we get

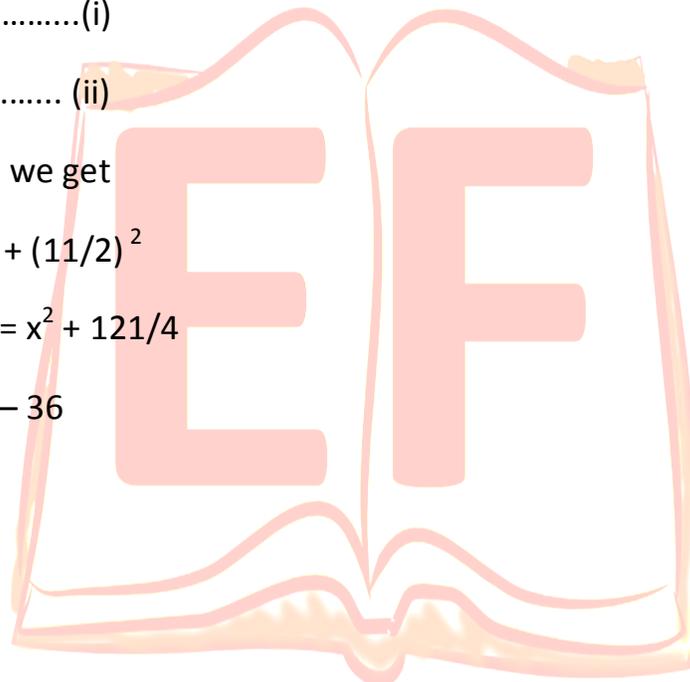
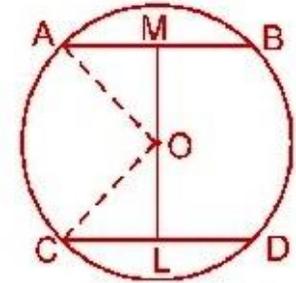
$$r^2 = (6 - x)^2 + (5/2)^2$$

$$\Rightarrow r^2 = (6 - 1)^2 + (5/2)^2$$

$$\Rightarrow r^2 = 5^2 + 25/4$$

$$\Rightarrow r^2 = 25 + 25/4$$

$$\Rightarrow r^2 = 125/4$$



$$\Rightarrow r = \sqrt{125/4}$$

$$\Rightarrow r = 5\sqrt{5}/2$$

Hence, radius $r = 5\sqrt{5}/2$ cm.

Question 3:

The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

Answer:

Let PQ and RS be two parallel chords of a circle with centre O.

We have, PQ = 8 cm and RS = 6 cm.

Draw perpendicular bisector OL of RS which meets PQ in M.

Since PQ || RS, therefore, OM is also perpendicular bisector of PQ.

Also, OL = 4 cm and RL = RS/2

$$\Rightarrow RL = 3 \text{ cm}$$

$$\text{and } PM = PQ/2$$

$$\Rightarrow PM = 4 \text{ cm}$$

In $\triangle ORL$, we have

$$OR^2 = RL^2 + OL^2 \quad [\text{Pythagoras theorem}]$$

$$\Rightarrow OR^2 = 3^2 + 4^2 = 9 + 16$$

$$\Rightarrow OR^2 = 25$$

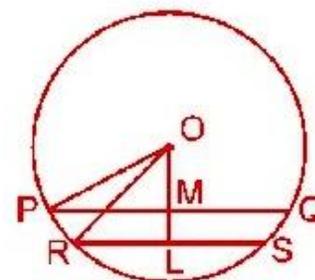
$$\Rightarrow OR = 5 \text{ cm}$$

Since $OR = OP$ [Radii of the circle]

$$\Rightarrow OP = 5 \text{ cm}$$

Now, in $\triangle OPM$

$$OM^2 = OP^2 - PM^2 \quad [\text{Pythagoras theorem}]$$



$$\Rightarrow OM^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$$\Rightarrow OM = \sqrt{9} = 3 \text{ cm}$$

Hence, the distance of the other chord from the centre is 3 cm.

Question 4:

Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Answer:

Given: Two equal chords AD and CE of a circle with centre O. When meet at B when produced.

To Prove: $\angle ABC = (\angle AOC - \angle DOE)/2$

Proof: Let $\angle AOC = x$, $\angle DOE = y$, $\angle AOD = z$

$\angle EOC = z$ [Equal chords subtends equal angles at the centre]

$$\Rightarrow x + y + 2z = 360^\circ \quad \text{[Angle at a point](i)}$$

$OA = OD$

$$\Rightarrow \angle OAD = \angle ODA$$

Now in $\triangle OAD$, we have

$$\angle OAD + \angle ODA + z = 180^\circ$$

$$\Rightarrow 2\angle OAD = 180^\circ - z \quad \text{[Since } \angle OAD = \angle ODA \text{]}$$

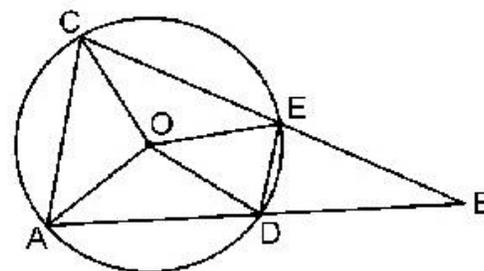
$$\Rightarrow \angle OAD = 90^\circ - z/2 \quad \text{..... (ii)}$$

Similarly $\angle OCE = 90^\circ - z/2$ (iii)

$$\Rightarrow \angle ODB = \angle OAD + \angle ODA \quad \text{[Exterior angle property]}$$

$$\Rightarrow \angle OEB = 90^\circ - z/2 + z \quad \text{[From equation (ii)]}$$

$$\Rightarrow \angle ODB = 90^\circ + z/2 \quad \text{..... (iv)}$$



Also, $\angle OEB = \angle OCE + \angle COE$ [Exterior angle property]

$\Rightarrow \angle OEB = 90^\circ - z/2 + z$ [From equation (iii)]

$\Rightarrow \angle OEB = 90^\circ + z/2$ (v)

Also, $\angle OED = \angle ODE = 90^\circ - y/2$ (vi)

From equation (iv), (v) and (vi), we have

$$\angle BDE = \angle BED = 90^\circ + z/2 - (90^\circ - y/2)$$

$\Rightarrow \angle BDE = \angle BED = 90^\circ + z/2 - 90^\circ + y/2$

$\Rightarrow \angle BDE = \angle BED = (y + z)/2$

$\Rightarrow \angle BDE = \angle BED = y + z$ (vii)

So, $\angle BDE = 180^\circ - (y + z)$

$\Rightarrow \angle ABC = 180^\circ - (y + z)$ (viii)

Now, $(y - z)/2 = (360^\circ - y - 2z - y)/2 = 180^\circ - (y + z)$ (ix)

From equation (viii) and (ix), we get

$$\angle ABC = (x - y)/2$$

Question 5:

Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

Answer:

Given: A rhombus ABCD whose diagonals intersect each other at O.

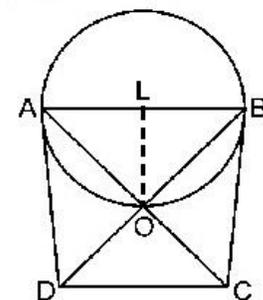
To prove: A circle with AB as diameter passes through O.

Proof: $\angle AOB = 90^\circ$ [Diagonals of a rhombus bisect each other at 90°]

$\Rightarrow \triangle AOB$ is a right triangle right angled at O. $\Rightarrow AB$ is the hypotenuse of $\triangle AOB$.

\Rightarrow If we draw a circle with AB as diameter, then it will pass through O. because angle in a

Semicircle is 90° and $\angle AOB = 90^\circ$



Question 6:

ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that $AE = AD$.

Answer:

Given: ABCD is a parallelogram.

To Prove: $AE = AD$.

Construction: Draw a circle which passes through ABC and intersect CD (or CD produced) at E.

Proof: For figure (i), we get

$$\angle AED + \angle ABC = 180^\circ \quad [\text{Linear pair}] \dots\dots\dots (i)$$

$$\text{But } \angle ACD = \angle ADC = \angle ABC + \angle ADE$$

$$\Rightarrow \angle ABC + \angle ADE = 180^\circ \quad [\text{From figure (ii)}] \dots\dots\dots (ii)$$

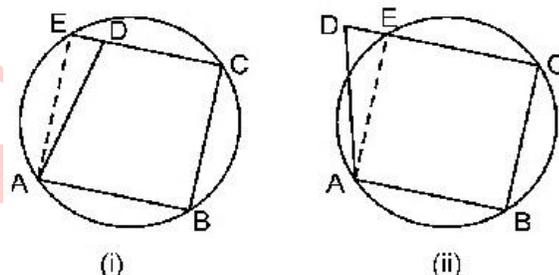
From equation (i) and (ii), we get

$$\angle AED + \angle ABC = \angle ABC + \angle ADE$$

$$\Rightarrow \angle AED = \angle ADE$$

$$\Rightarrow \angle AD = \angle AE \quad [\text{Sides opposite to equal angles are equal}]$$

Similarly we can prove for Fig (ii) also.



Question 7:

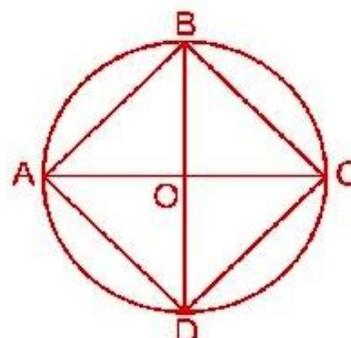
AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is a rectangle.

Answer:

Given: A circle with chords AB and CD which bisect each other at O.

To Prove: (i) AC and BD are diameters

(ii) ABCD is a rectangle.



Proof: In ΔOAB and ΔOCD , we have

$$OA = OC \quad [\text{Given}]$$

$$OB = OD \quad [\text{Given}]$$

$$\angle AOB = \angle COD \quad [\text{Vertically opposite angles}]$$

$$\Rightarrow \Delta AOB \cong \Delta COD \quad [\text{SAS congruence}]$$

$$\Rightarrow \angle ABO = \angle CDO \text{ and } \angle BAO = \angle BCO \quad [\text{CPCT}]$$

$$\Rightarrow AB \parallel DC \quad \dots\dots\dots (i)$$

Similarly, we can prove $BC \parallel AD \quad \dots\dots\dots (ii)$

Hence, ABCD is a parallelogram.

But ABCD is a cyclic parallelogram.

So, ABCD is a rectangle. [Proved in Q. 12 of Ex. 10.5]

$$\Rightarrow \angle ABC = 90^\circ \text{ and } \angle BCD = 90^\circ$$

$$\Rightarrow AC \text{ is a diameter and } BD \text{ is a diameter} \quad [\text{Angle in a semicircle is } 90^\circ]$$

Question 8:

Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90^\circ - A/2$, $90^\circ - B/2$ and $90^\circ - C/2$.

Answer:

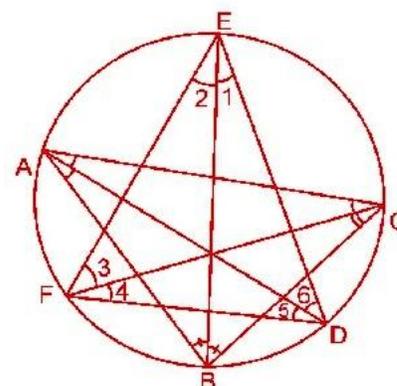
Given : ΔABC and its circumcircle. AD, BE, CF are bisectors of $\angle A$, $\angle B$, $\angle C$ respectively.

Construction : Join DE, EF and FD.

Proof : We know that angles in the same segment are equal.

$$\text{So, } \angle 5 = \angle C/2 \text{ and } \angle 6 = \angle B/2 \quad \dots\dots\dots (i)$$

$$\angle 1 = \angle A/2 \text{ and } \angle 2 = \angle C/2 \quad \dots\dots\dots (ii)$$



$$\angle 4 = \angle A/2 \text{ and } \angle 3 = \angle B/2 \dots\dots\dots\text{(iii)}$$

From equation (i), we have

$$\angle 5 + \angle 6 = \angle C/2 + \angle B/2$$

$$\Rightarrow \angle D = \angle C/2 + \angle B/2 \dots\dots\dots\text{(iv)} \quad [\text{Since } \angle 5 + \angle 6 = \angle D]$$

But $\angle A + \angle B + \angle C = 180^\circ$

$$\Rightarrow \angle B + \angle C = 180^\circ - \angle A$$

$$\Rightarrow \angle B/2 + \angle C/2 = 90^\circ - \angle A/2$$

So, equation (iv) becomes,

$$\angle D = 90^\circ - \angle A/2$$

Similarly, from equation (ii) and (iii), we can prove that

$$\angle E = 90^\circ - \angle B/2 \text{ and } \angle F = 90^\circ - \angle C/2$$

Question 9:

Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Answer:

Given: Two congruent circles which intersect at A and B. PAB is a line through A.

To Prove: BP = BQ.

Construction: Join AB.

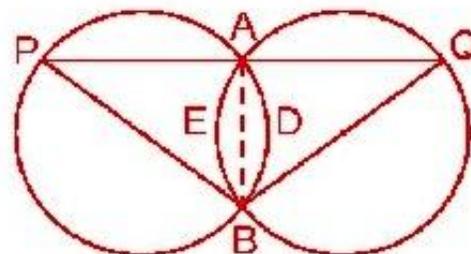
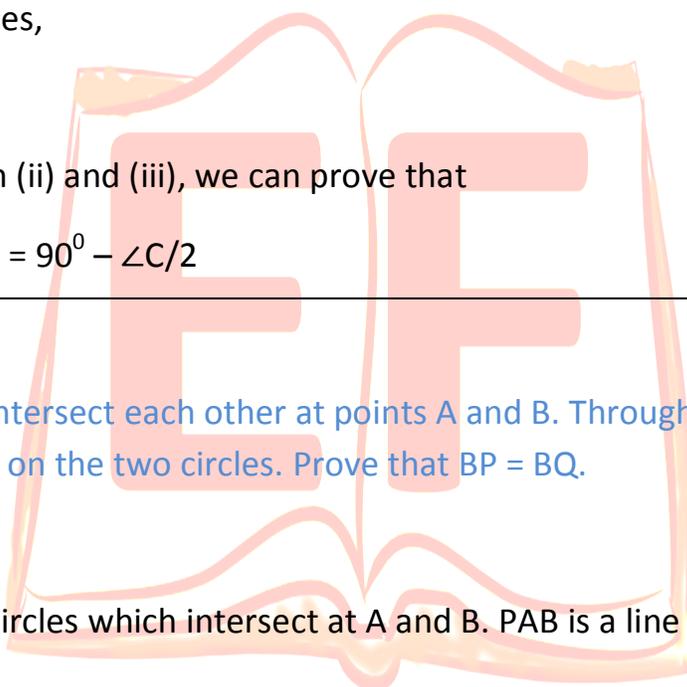
Proof: AB is a common chord of both the circles.

But the circles are congruent.

$$\Rightarrow \text{arc ADB} = \text{arc AEB}$$

$$\Rightarrow \angle APB = \angle AQB \quad [\text{Angles subtended}]$$

$$\Rightarrow BP = BQ \quad [\text{Sides opposite to equal angles are equal}]$$



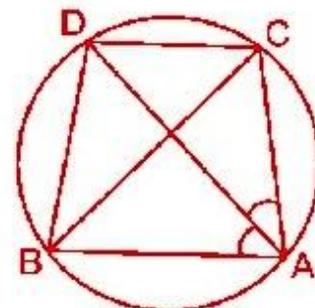
Question 10:

In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

Answer:

Let angle bisector of $\angle A$ intersect circumcircle of ΔABC at D.

Now, join DC and DB.



$$\angle BCD = \angle BAD \quad [\text{Angles in the same segment}]$$

$$\Rightarrow \angle BCD = \angle BAD = \angle A/2 \quad [\text{AD is bisector of } \angle A] \dots\dots(i)$$

$$\text{Similarly } \angle DBC = \angle DAC = \angle A/2 \dots\dots(ii)$$

From equation (i) and (ii), we get

$$\angle DBC = \angle BCD$$

$$\Rightarrow BD = DC \quad [\text{sides opposite to equal angles are equal}]$$

\Rightarrow D lies on the perpendicular bisector of BC.

Hence, angle bisector of $\angle A$ and perpendicular bisector of BC intersect on the circumcircle of ΔABC .